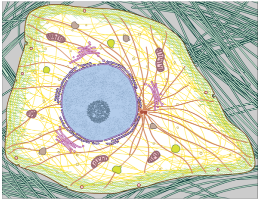
"MP Grammars, Reactive Systems and Electric Circuits"
(Thesis Proposal)

Ricardo Henrique Gracini Guiraldelli

Università degli Studi di Verona

November 14, 2013

The marketing (or general idea for the layman)



Source: The Cell: An Image Library. http://www.cellimagelibrary.org/pages/cell_illustrations.

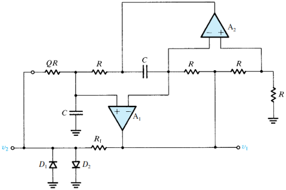
$$\stackrel{?}{\equiv}$$

$$\stackrel{?}{\Leftrightarrow}$$

$$\stackrel{?}{\approx}$$

$$\stackrel{?}{\models}$$

$$\vdots$$



Source: Sedra, A.S., Smith K.C.: Microeletronic Circuits. Oxford University Press (2009).

The marketing (or general idea for the layman)

$$\xi(Cell) = Circuit$$

$$\zeta(Circuit) = Cell$$

The marketing (or general idea for the layman)

$$\xi(\mathit{Cell}) = \mathit{Circuit}$$

$$\exists \xi, \zeta ?\ \zeta = \xi^{-1} ?$$

$$\zeta(\mathit{Circuit}) = \mathit{Cell}$$

How to (mathematically) model a cell?

- Cell is a black box.

$$\{I_1, I_2, \ldots\} \rightarrow \boxed{\text{C}} \rightarrow \{O_1, O_2, \ldots\}$$

- It does not have a (standard) mathematical representation.
- It is composed of a series of other process.
  - Many of them are *unknown*.
  - One does not know the *order of execution* of them.
- Fundamental computational unit of Life.

Metabolic P system

- Discrete dynamical system.
- Based on:
  - formal languages;
  - recurrence equations;
  - linear algebra;
  - statistics;
  - membrane computing.
- Designed to model metabolic process, but can model a wide range of process.
- Very expressive:
  - matrix and recurrence equations supported *grammar notation*;
  - *graph*(ical) *representation*.

MP system in the wild

Prey-predator or Lotka-Volterra model

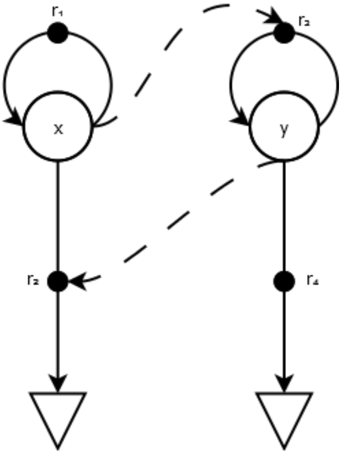
The Lotka-Volterra model is the simplest prey-predator interaction model, presented as an oscillating model. The prey and predator population is represented, respectively, by the variables x and y; four constants, A, B, C and D, represent the reproduction factor of both species, the predation rate and the loss of predators.
A pair of ODEs models its dynamics as following:

$$\begin{aligned} \frac{dx}{dt} &= (A - By)x \\ \frac{dy}{dt} &= (Cx - D)y \end{aligned}$$

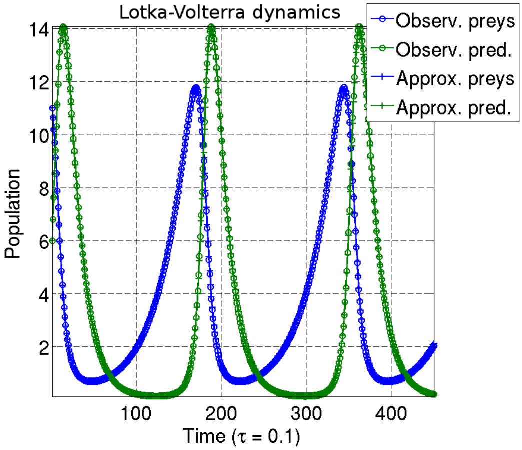
MP system in the wild

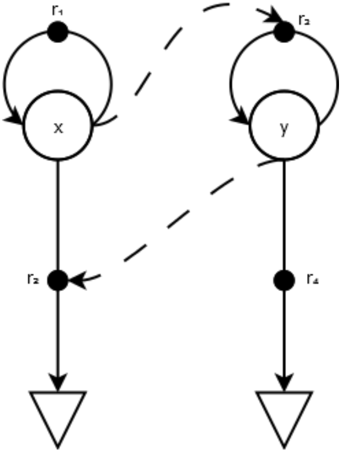
Rules	**Regulators**
$r_1 : x \to 2x$	$\phi_1 = 3 \times 10^{-2} x$
$r_2 : x \to \emptyset$	$\phi_2 = 9 \times 10^{-4} + 9 \times 10^{-3} xy + 10^{-4} x^2 y$
$r_3 : y \to 2y$	$\phi_3 = 9 \times 10^{-4} + 1.5 \times 10^{-2} xy + 3 \times 10^{-4} x^2 y$
$r_4 : y \to \emptyset$	$\phi_4 = 6.6 \times 10^{-2} y$

$$
\begin{aligned}
U[i] &= \left(\phi_1(i), \phi_2(i), \ldots, \phi_m(i)\right)^T \\
\Delta[i] &= \mathbb{A} \times U[i] \\
X[i+1] &= X[i] + \Delta[i]
\end{aligned}
$$



MP system in the wild





Source: Manca, V. Infobiotics: Information in Biotic Systems. Springer Berlin Heildelberg (2013).

Dynamical inverse problem

- MP system is very expressive but... How should I know the declaration of the ϕ's?

Dynamical inverse problem

- MP system is very expressive but... How should I know the declaration of the ϕ's?

$$\text{Recall: } \{I_1, I_2, \ldots\} \rightarrow \boxed{\text{C}} \rightarrow \{O_1, O_2, \ldots\}$$

Dynamical inverse problem

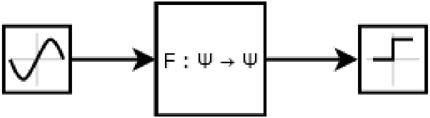
- MP system is very expressive but... How should I know the declaration of the ϕ's?

$$\text{Recall: } \{I_1, I_2, \ldots\} \rightarrow \boxed{\text{C}} \rightarrow \{O_1, O_2, \ldots\}$$

- Log-Gain Stepwise Regression.
  - Input: time series of input and output values; regressors dictionary; rules.
  - Output: regulators.
- LGSS is a heavy and non-trivial algorithm, but there is a Java library to take care of it.

Signals, Systems and Circuits

- Signal: information carrier. ψ : *Time* $\mapsto$ *Value*.
- Systems: process that generates or modifies signals.
  $F : \Psi \mapsto \Psi \mid \Psi = \{\psi_1, \psi_2, \ldots\}$.



Systems and their properties

- Generally represented by *block diagrams*.
- Composable—just as functions. Even easier in block representation than functional one.
- Two big classification:
  - Transducive systems. $F_T : \mathit{Value} \mapsto \mathit{Value}$.
  - Reactive systems. $F_R : \Psi \mapsto \Psi$.

Note: $F_R \models F_T$.

Particular reactive systems

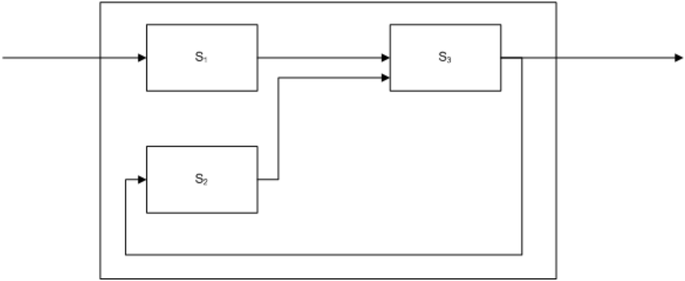
Delay systems Those in which the outputted values are transformations over "delayed inputs", i.e.,

$$y(t_i) = \begin{cases} \kappa & \text{if } t_i < \delta \\ f(x(t_i - \delta)) & \text{if } t_i \geq \delta \end{cases}$$

where κ is a constant value defined by the system.

Feedback systems Kind of delay systems in which the input data $x(t_{i+\delta})$ is (composed by) the output $y(t_i)$.

Particular reactive systems



Electrical circuits, the well-known model

- Mathematical model of *physical* electrical circuits.
- Simplifies the circuit analisys—keep Maxwell('s laws) away!
- Classified according to the type of signal they process: *analog vs. digital*.
    - Components also changes.
    - Discretization of signal transform from analog to digital.

Once upon a time...

- Terje Lømo and long-term potentiation: shocks on rabbits make them "smart"!
  - Series of electrical stimulation in the rabbits.
  - Series of electrical response from the rabbits brain.
  - Work started in the 1960's... Lot unknown.
  - Response curves seems transformed by phase, amplitude, etc. *Remind me the Electrical Circuit classes*.
  - What if we apply LGSS to these data?

Once upon a time...

- Vincenzo Manca and Luca Marchetti: *"actually, we've been thinking something similar..."*
  - What if we model as circuits those projects we are working in?
  - What if we could used reduction techniques from harware synthesis into MP grammars?
  - What if we could work with analog data on MP? Would it be an analog computer?

Once upon a time...

- Vincenzo Manca and Luca Marchetti: *"actually, we've been thinking something similar..."*
  - What if we model as circuits those projects we are working in?
  - What if we could used reduction techniques from harware synthesis into MP grammars?
  - What if we could work with analog data on MP? Would it be an analog computer?
- "Eureka": is there an equivalence between MP system and electric circuits?

$\exists$idea | idea = mine?

- MP systems belongs to membrane computing.
- There are efforts to implement models of membrane computing in hardware.
  - However, based on other models.
  - Concerned to find unconventional computer architectures.
- Control theory approach is the most similar to the current proposal.
  - In fact, some inspiration come from this field.

The proposal is...

Let Ψ_I be a set of indexed input signals and Ψ_O a set of output indexed signals.

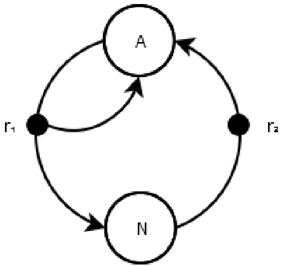
- For all MP system $\mathcal{M}$ that produces Ψ_O given Ψ_I, does exist an electric circuit $\mathcal{C}$ that also produces Ψ_O given Ψ_I?
- For all electric circuit $\mathcal{C}$ that produces Ψ_O given Ψ_I, does exist a MP system $\mathcal{M}$ that also produces Ψ_O given Ψ_I?
- Is there a transformation $\xi(\mathcal{M}) = \mathcal{C}$?
- Is there a transformation $\zeta(\mathcal{C}) = \mathcal{M}$?
- If those transformations exist, how do they relate to each other?

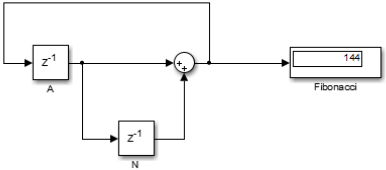
Following the intuition

$$\begin{cases} Fib(0) = 1 \\ Fib(1) = 1 \\ Fib(n) = Fib(n-1) + Fib(n-2) \end{cases}$$

Rules	**Regulators**
$r_1 : A \rightarrow A + N$	$\phi_1 = A$
$r_2 : N \rightarrow A$	$\phi_2 = N$

Following the intuition





Divide and conquer

- Electric circuits is a broad and complex field.
- "Low-hanging fruits" strategy: focus on the easy-to-reach results.
- MP system $\rightarrow$ reactive systems $\rightarrow$ digital circuit first.
  - Explore existing knowledge: signals and systems, control theory, computer-aided design, hardware description languages, etc...
  - Boolean networks.
  - Hardware-software equivalence.
- MP system $\rightarrow$ analog circuits.
  - Discretize circuits vs. "analogize" MP systems.
  - Map circuits as regressors—and verify its "satisfability of functions"
  - Broader field.

Last words—or why does it worth?

- MP systems are successfully applied in a range of fields: mathematics, chemistry, biology, etc... But "what about the neighbours", eletronics?
- Exchange of knowledge may enrich both fields, as exists in math-physics, physics-chemistry, chemistry-biology, etc...
- "Cell-on-a-chip": FPGA- or chemistry kit-like solution for biologists and physicians!

The end.

Thank you!

Grazie!

Obrigado!

Ačiū!